Hierarchical ordering of reticular networks

The structure of hierarchical networks in biological and physical systems has long been characterized using the Horton-Strahler ordering scheme. The scheme assigns an integer order to each edge in the network based on the topology of branching such that the order increases from distal parts of the network (e.g., mountain streams or capillaries) to the “root” of the network (e.g., the river outlet or the aorta). However, Horton-Strahler ordering cannot be applied to networks with loops because they create a contradiction in the edge ordering in terms of which edge precedes another in the hierarchy. In this talk I will present a generalization of the Horton-Strahler order to weighted planar reticular networks, where weights are assumed to correlate with the importance of network edges, e.g., weights estimated from edge widths may correlate to flow capacity. The new method assigns hierarchical levels not only to edges of the network, but also to its loops, and classifies the edges into reticular edges, which are responsible for loop formation, and tree edges. I will show that the sensitivity of the hierarchical levels to weight perturbations can be analyzed in a rigorous way. I will also discuss applications of this generalized Horton-Strahler ordering to the study of leaf venation and other biological networks.